(Pages : 4) **P – 2487**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1544 : DIFFERENTIAL EQUATIONS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

PART – A

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Define order and degree of a ordinary differential equation.

2. Solve $\frac{dy}{dx} = 4y$ and $y(0) = 2$.

3. Show that the differential equation $(y^2 - \cos^2 x)\frac{dy}{dx} = y \sin 2x$ is exact.

4. Solve the differential equation $\frac{dy}{dx} = xy$.

5. Find the integrating factor of the differential equation $\frac{dy}{dx} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$.

6. Write the general form of Euler – Cauchy equation.

7. Find the complementary function of the differential equation $\frac{d^2y}{dx^2}+4=8\cos x$.

8. Solve the second order differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}=4x$ to find $\frac{dy}{dx}$.

9. Show that $y_1 = e^x$ and $y_2 = xe^x$ are Linearly independent functions.

10. Define Basis of solutions of a differential equation $y''+P(x)y'+Q(x)y=0$.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. Each question carries **2** marks.

11. Solve the differential equation $\frac{dy}{dx}+2xy=4x$.

12. Solve $3e^x \tan y+(1-e^x)\sec^2 y\frac{dy}{dx}=0$. .

13. Find the integral factor of the differential equation $\frac{dy}{dx}+y\tan x=\cos^3 x$.

14. Solve the differential equation $xy'+y=0$.

15. Find the solution of the initial value problem $y'=3x^2e^{-y}$ and $y(0)=2$.

16. Solve the differential equation $\frac{dy}{dx}=x+y+1$.

17. Write a short note on a first-order linear and non-linear ordinary differential equations.

18. Write the general form of Bernoulli equation.

19. Solve the differential equation $y''+y=0$.

20. Solve $y''-y=x$.

21. Find the solution of the second order ODE, $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=0$.

22. Find a particular integral for $y'' - 3y' + 2y = e^x$.

23. Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

24. Write a differential equation of the form $y''+ay'+by = 0$ for which the functions e^{7x} and e^{4x} form a basis.

25. Find a real general solution of $x^2y''-20y = 0$.

26. Solve the differential equation $(D^2 + 4D + 4)y = 0$.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. **Each** question carries **4** marks.

27. Solve $\cos(x+y)dx + (3y^2 + 2y + \cos(x+y))dy = 0$.

28. Solve the differential equation $\frac{dy}{dx} = -\frac{2}{y} - \frac{3y}{2x}$.

29. Solve the differential equation $(ax + hy + g)dx + (hx + by + f)dy = 0$.

30. Solve the differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$.

31. Solve the Bernoulli's equation $\frac{dy}{dx} + \frac{y}{x} = 2x^3y^4$.

32. Solve Differential equation $\frac{dy}{dx} + \frac{y}{x} = y^2 \log x$

33. Solve the second order ordinary differential equation $(D^2 + 4)y = \sin^2 x$.

34. Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + \frac{2}{y^3} = 0$.

35. Solve the differential equation $4x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2 - 1)y = 0$.

36. Solve $x^3\dfrac{d^2y}{dx^2}-\left(x^3+xy\right)\dfrac{dy}{dx}+\left(y^2+xy\right)=0$.

37. Solve the initial value problem $y''+y=0.001x^2; y(0)=0, y'(0)=1.5$.

38. Using the method of variation of parameters find the particular integral of $y''+y=\sec x$.

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions. **Each** question carries **15** marks.

39. (a) Solve the equation $\left(x^2-3y^2\right)dx+2xydy=0$.

(b) Solve the Bernoulli differential equation, $\dfrac{dy}{dx}+y=xy^3$.

40. (a) Solve the initial-value problem $\left(2x\cos y+3x^2y\right)dx+\left(x^3-x^2\sin y-y\right)dy-0$; $y(0)=2$.

(b) Solve the differential equation $\dfrac{dy}{dx}=\dfrac{2x-5y+3}{2x+4y-6}$.

41. Solve the initial-value problem $\left(y+\sqrt{x^2+y^2}\right)dx-xdy=0,\quad y(1)=0$.

42. (a) Solve the initial-value problem $y''-y'-12y=0,\ y(0)=3,\ y'(0)=5$.

(b) Solve the differential equation, $y^2dx+(3x-1)dy=0$.

43. (a) Use the variation-of-parameters method to solve $\dfrac{d^2y}{dx^2}+y=\operatorname{cosec} x$ subject to the boundary conditions $y(0)=y(\pi/2)=0$.

(b) Find the general solution of the differential equation $y''+4y=8\cos 2x$.

44. (a) Solve the initial value $y''+3y'+2.25y=-10e^{-1.5x},\ y(0)=1,\ y'(0)=0$.

(b) Find the solution of the Homogeneous Linear Equation $x^2y''+2xy'-20y=x^4$.

(2 × 15 = 30 Marks)

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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1541 : REAL ANALYSIS I

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION I

Answer **all** the questions.

1. Define absolute value function $|x|$.
2. Define *least upper bound* for a set $A \subseteq R$.
3. Find $\inf(A)$ if $A = \left\{\frac{1}{n}, n \in \mathbb{N}\right\}$
4. Find $\lim_{n\to\infty}\left(\frac{3n+2}{2n-2}\right)$.
5. Define the limit of a real sequence.
6. Write the harmonic series. Is it convergent?
7. Define Cauchy sequence.
8. Let $\epsilon = \frac{1}{2}$ and $a = 3$, then find the ϵ - neighborhood, $V_\epsilon(a)$ of a.
9. Define closed set in $\mathbb{R}$. Write an example for closed set.
10. Define perfect set in $\mathbb{R}$. **(10 × 1 = 10 Marks)**

P.T.O.

SECTION II

Answer any **eight** questions

11. Show that, two real numbers a and b are equal if and only if for every real number $\in>0$ it follows that $|a-b|<\in$.

12. Prove that $\sqrt{3}$ is irrational.

13. Using triangle inequality prove the inequality $||a|-|b|| \le |a-b|$.

14. Let $A \subseteq R$ be bounded above, and let $c \in R$. Define the set $c+A$ by $c+A=\{c+a : a \in A\}$. Show that $\sup(c+A)=c+\sup A$.

15. Show that $\lim\left(\frac{n+1}{n}\right)=1$.

16. Show that every convergent sequence is bounded.

17. Let $\lim a_n = a$, and $\lim b_n = b$, then prove that $\lim(a_n + b_n) = a + b$.

18. Show that if $(b_n) \to b$, then the sequence of absolute values $|b_n|$ converges to $|b|$.

19. Write the examples for
    (a) Sequences (x_n) and (y_n), which both diverge, but whose sum $(x_n + y_n)$ converges;
    (b) A convergent sequence (b_n) with $b_n \neq 0$ for all n such that $(1/b_n)$ diverges.

20. Prove that the sequence defined by $x_1 = 3$ and $x_{n+1} = \frac{1}{4-x_n}$ converges.

21. Show that, sub sequences of a convergent sequence converge to the same limit as the original sequence.

22. Show that, every convergent sequence is a Cauchy sequence.

23. Let $a \in A$. Prove that a is an isolated point of A if and only if there exists an $\in-$ neighborhood $V_\in(a)$ such that $V_\in(a) \cap A = \{a\}$.

24. Show that if K is compact, then $\sup K$ and $\inf K$ both exist and are elements of K.

25. Prove that cantor set is a Compact set.

26. Let E is nowhere-dense in $\mathbb{R}$, then show that the complement of $\overline{E}$ is dense in $\mathbb{R}$.

(8 × 2 = 16 Marks)

SECTION III

Answer any **six** questions

27. Assume $s \in R$ is an upper bound for a set $A \subseteq R$. Then, show that $s = \sup A$ if and only if, for every choice of $\in > 0$, there exists an element $a \in A$ satisfying $s - \in < 0$.

28. State and prove Nested Interval Property.

29. Show that the set of rational numbers, $\mathbb{Q}$ is countable.

30. Given any set A, prove that there does not exist a function $f : A \to P(A)$ that is onto.

31. Assume $\lim a_n = a$ and $\lim b_n = b$, = b, then show that

    (a) If $a_n \geq 0$ for all $n \in \mathbb{N}$, then $a \geq 0$.

    (b) If $a_n \leq b_n$ for all $n \in \mathbb{N}$, then $a \leq b$.

32. Show that; if $x_n \leq y_n \leq z_n \forall n \in \mathbb{N}$, and if $\lim x_n = \lim z_n = l$, then $\lim y_n = l$.

33. If a sequence is monotone and bounded, then show that it is convergent.

34. Show that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.

35. Prove that, a point x is a limit point of a set A if and only if $x = \lim a_n$ for some sequence (a_n) contained in A satisfying $a_n \neq x$ for all $n \in \mathbb{N}$.

36. For any $A \subseteq \mathbb{R}$, Show that the closure $\overline{A}$ is a closed set and is the smallest closed set containing A.

37. If $K_1 \supseteq K_2 \supseteq K_3 \supseteq K_4 \supseteq \ldots$ is a nested sequence of nonempty compact sets, then show that, the intersection $\bigcap_{n=1}^{\infty} K_n$ is not empty.

38. Prove that the set of real numbers $\mathbb{R}$ cannot be written as the countable union of nowhere-dense sets.

(6 × 4 = 24 Marks)

SECTION IV

Answer any **two** questions

39. Prove that

(a) Given any number $x \in \mathbb{R}$, there exists an $n \in \mathbb{N}$ satisfying $n > x$.

(b) Given any real number $y > 0$, there exists an $n \in \mathbb{N}$ satisfying $\frac{1}{n} < y$.

(c) For every two real numbers a and b with $a < b, \exists$ a rational number r, satisfying $a < r < b$.

40. (a) Show that, there exists a real number $\alpha \in \mathbb{R}$ satisfying $\alpha^2 = 2$.

(b) Find *suprema* and *infima* of the following sets.

(i) $\{n \in \mathbb{N} : n^2 < 10\}$

(ii) $\{\frac{n}{m} : m, n \in \mathbb{N}$ with $m + n \leq 10\}$

41. (a) State and prove Bolzano-Weierstrass theorem.

(b) Prove or disprove: Every Bounded sequence of real number are convergent.

42. Show that

(a) Cauchy sequences are bounded.

(b) A sequence converges if and only if it is a Cauchy sequence.

43. (a) State and prove Cauchy Criterion for Series.

(b) Using Alternating Series Test, test the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{7+2n}$.

44. (a) Show that

(i) The union of an arbitrary collection of open sets is open.

(ii) The intersection of a finite collection of open sets is open.

(b) Let $B = \left\{ \frac{(-1)^n n}{n+1}, n \in \mathbf{N} \right\}$

(i) Find the limit points of B.

(ii) Does B contain any isolated points?

(iii) Find closure of B, $\overline{B}$.

(2 × 15 = 30 Marks)

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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1543 : ABSTRACT ALGEBRA – GROUP THEORY

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION I

Answer **all the questions.**

1. Define Reflexive and transitive properties of a relation R on a set.
2. Let $\mathbb{N}$ represent the set of natural numbers and + represent usual addition, verify whether the structure $(\mathbb{N}, +)$ form a group or not?
3. Show that the identity element in a group is unique.
4. Find inverse of 13 in $(\mathbb{Z}_{20}, +_{20})$.
5. Write the permutation (1235) (413) as a product of disjoint cycles.
6. Find the order of the following permutation (1 4).
7. Define Group Isomorphism.
8. Consider the group $\mathbb{Z}_4$, *under addition modulo* 4. Let $H = \{0, 2\}$ be a subgroup. Then find all left cossets of H.
9. Define Normal subgroup.
10. Define Kernel of a group Homomorphism.

(10 × 1 = 10 Marks)

P.T.O.

SECTION II

Answer any **eight** questions.

11. Verify whether the set of integers form a group under usual multiplication or not?

12. Write an example for non-abelian group.

13. Define center of a group with example.

14. Prove that in any group, an element and its inverse have the same order.

15. If a and b are group elements and $ab \neq ba$ prove that $aba \neq e$.

16. Find all generators of Z_6 and Z_8.

17. Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$ = Find $\beta\alpha$ and $\alpha\beta$.

18. Find an isomorphism from the group of integers under addition to the group of even integers under addition.

19. Prove that S_4 is not isomorphic to D_{12}.

20. Let G be the real numbers under addition and let $\overline{G}$ be the positive real numbers under multiplication. Then show that $G \approx \overline{G}$.

21. Suppose that ϕ is an isomorphism from a group G onto a group $\overline{G}$. Then show that ϕ^{-1} is an isomorphism from $\overline{G} \to G$.

22. Prove that the set of automorphisms of a group and the set of inner automorphisms of a group are both groups under the operation of function composition.

23. Show that, for each element a in a group G, there is a unique element b in G such that $ab = ba = e$.

24. Let $a, b \in G$, where G is a group, then show that $(ab)^{-1} = b^{-1}a^{-1}$.

25. Let G be a group and H a nonempty subset of G. If $ab^{-1} \in H$ whenever a and b are in H, then Show that H is a subgroup of G.

26. Show that, the *center* of a group G is a subgroup of G.

(8 × 2 = 16 Marks)

SECTION III

Answer any **six** questions.

27. Show that the set of n-tuples over $\mathbb{R}$, $\mathbb{R}^n = \{(x_1, x_2, \ldots \ldots x_n) : x_1, x_2, \ldots \ldots x_n \in \mathbb{R}\}$ form an abelian group under component wise addition.

28. Let H be a nonempty finite subset of a group G. If H is closed under the operation of G, then show that H is a subgroup of G.

29. State and prove fundamental theorem of cyclic groups.

30. Show that every group is isomorphic to a group of permutations.

31. Let H be a subgroup of G, and let a and b belong to G. Then prove that $aH = Ha$ if and only if $H = aHa^{-1}$.

32. Show that, a group of prime order is cyclic.

33. Prove that a group of order 12 must have an element of order 2.

34. Prove that a subgroup H of G is normal in G if and only if $xHx^{-1} = H, \forall x$ in G.

35. Let G be a group and let H be a normal subgroup of G, then show that the set $G/H = \{aH \mid a[G\}$ is a group under the operation $(aH)(bH) = abH$.

36. Let G be a group and let $Z(G)$ be the center of G. If $G/Z(G)$ is cyclic, then show that G is Abelian.

37. Suppose that $\phi : Z_{50} \to Z_{15}$ is a group homomorphism with $\phi(7) = 6$. Find $\phi(x)$.

38. Find all Abelian groups (up to isomorphism) of order 360.

(6 × 4 = 24 Marks)

SECTION IV

Answer any **two** questions.

39. (i) Define

(a) a Group,

(b) Abelian group

(c) Cyclic group

(ii) Let G is the set of all 2×2 matrices whose determinant is non zero. Prove that G form a group under matrix multiplication. Is this group Abelian? Justify?

(iii) Let G is the set of all 2×2 matrices. Is G a group under matrix multiplication?

40. (i) Let G be a group, and let $a \in G$. Then, show that $\langle a \rangle$ is a subgroup of G.

(ii) Define the Centralizer of an element in a group G.

(iii) Show that for each a in a group G, the centralizer of a is a subgroup of G.

41. Let $G = \{(1), (132)(465)(78), (132)(465), (123(456), (123)(456)(78), (78)\}$

Then find (i) Orbit of 1, 2, 4 & 7

(ii) Stabilizer of 1, 2, 4 & 7

42. (i) State and prove first Isomorphism theorem.

(ii) If ϕ is a homomorphism from a finite group G to $\overline{G}$, then $|\phi(G)|$ divides $|G|$ & $|\overline{G}|$.

43. (i) Prove that, every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

(ii) If the pair of cycles $\alpha = (a_1, a_2, \ldots, a_m)$ and $\beta = (b_1, b_2, \ldots, b_n)$ have no entries in common, then show that $\alpha\beta = \beta\alpha$.

44. (i) Let $\mathbb{Z} = \{\ldots \ldots -2, -1, 0, 1, 2, \ldots \ldots\}$ and $4\mathbb{Z} = \{\ldots \ldots -8, -4, 0, 4, 8, \ldots \ldots\}$ then construct the factor group $\mathbb{Z}/4\mathbb{Z}$.

(ii) Let G be a finite Abelian group and let p be a prime that divides the order of G. Then prove that G has an element of order p.

(2 × 15 = 30 Marks)

(Pages : 4) **P – 2490**

Reg. No. : ....

Name : ....

Fifth Semester B.A./B.Sc./B.Com. Degree Examination, December 2022

First Degree Programme under CBCSS

Mathematics

Open Course

MM 1551.3 : BASIC MATHEMATICS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define improper fractions.

2. *Simplify* $10-(1-3)^2 2^3$.

3. State the divisibility rule for dividing by 3.

4. Determine the place value of 3 in 547.398.

5. Convert $3\frac{3}{21}$, into an improper fraction.

6. Write 123.45 as a fraction.

P.T.O.

7. Find $\frac{2}{9}+\frac{7}{5}$.

8. Find the mean of the first 10 even numbers.

9. Define mode.

10. Define an obtuse triangle.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. **Each** question carries **2 marks.**

11. Find $\frac{3}{7}\times\frac{9}{2}-\frac{4}{5}$.

12. Convert $22\frac{32}{25}$ into an improper fraction...

13. Convert $\frac{73}{25}$ into a mixed number.

14. Find the median of the first 10 prime numbers.

15. Simplify $17 + 3(7-\sqrt{9})^2$.

16. Find the prime factorisation of 320.

17. Find the LCM of 17 and 19.

18. Write two equivalent fractions of $2/7$.

19. Find the decimal equivalent of $34/99$.

20. Find $2\frac{4}{5}\div 3\frac{2}{7}$.

21. Convert the fraction $3/7$ to decimal form and then to percent form.

22. Solve $x^2 - 7x + 12 = 0$.

23. Define logarithm of a number.

24. Evaluate $\sqrt{125} \times \sqrt{20}$.

25. Find the determinant of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

26. Find the adjoint of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. **Each** question carries **4** marks.

27. Simplify $\sqrt[3]{\frac{32}{9} \div \frac{3}{2}} \times \frac{3}{4} + \sqrt{\sqrt{\frac{128}{27} \div \frac{3}{2}} + 1} \times \frac{3}{5}$.

28. Describe histograms with an example.

29. Find the weighted arithmetic mean of the first 10 even numbers with the first ten odd numbers as the weights.

30. Find the LCM and GCD of 36 and 60.

31. State the three laws of exponents.

32. State the three laws of logarithms.

33. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

34. What percent of 320 is 20? Express as a decimal after rounding.

35. Solve the quadratic equation $2x^2 - 7x + 5 = 0$.

36. Solve the simultaneous equations $2x+3y=12$ and $3x+2y=13$.

37. Calculate the total simple interest on a loan of Rs. 10,000 at 5% annual interest after 5 years and 6 months. Also find the total amount to be paid.

38. Construct a histogram for the frequency of prime numbers up to 25, with classes of size 5: The classes are 1-5, 6-10,...,21-25, while the respective frequencies are the number of primes among 1-5, number of primes among 6-10,.., number of primes among 21-25.

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions. **Each** question carries **15** marks.

39. Define mean, median, mode and weighted mean of a data set, illustrating each with an example. Calculate all of them for the first 10 prime numbers. For calculating the weighted mean, take the whole numbers 1 to 10 as the weights.

40. A bank offers loans under schemes. One, at a compound interest rate of 10% annually; Another, at a simple interest of 15% annually. Which of the two schemes is beneficial if you need to take a loan of Rs. 1,00,000 for 3 years? Does the answer change depending on the loan amount or the loan period?

41. Solve the system of equations by finding the inverse of the matrix:

$$x+2y+3z=6$$
$$2x+3y+z=6.$$
$$3x+y+2z=6$$

42. Solve the system of equations by Cramer's Rule:

$$x+y+z=3$$
$$x+2y+3z=6$$
$$2x+y+4z=7$$

43. Describe the method to solve a general quadratic equation, stating the role of its discriminant. Solve the quadratic equation $5x^2+10x-15=0$.

44. Describe a Geometric Progression with example. Derive the expression for the sum of its first n terms.

(2 × 15 = 30 Marks)

(Pages : 4) **P – 2485**

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1542 : COMPLEX ANALYSIS I

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** the questions.

1. Write $\frac{8i-1}{i}$ in the form of $a+ib$.
2. Find $(i)^{62}$.
3. Which of the points $-i$, $2i$, $1-2i$, -3 is farthest from origin?
4. Write Euler's formula.
5. Give an example of a bounded set and unbounded set.
6. Describe the range of $f(z)=z+5$, $\operatorname{Re}(z)>0$.
7. Define analytic function and give an example.
8. State the fundamental theorem of Algebra.
9. Write z^4-16 in factored form.
10. Find $\int_C (3z^2-5z+i)dz$ along the line segment from $z=i$ to 1.

(10 × 1 = 10 Marks)

P.T.O.

PART – B

Answer any **eight** questions.

11. Find the quotient $\frac{(6+2i)-(1+3i)}{(-1+i)-2}$.

12. Show that $\text{Re}(iz) = -\text{Im}(z)$ for any complex number z.

13. Express $1+\sqrt{3}i$ in polar form and hence find $\arg(1+\sqrt{3}i)$.

14. State and prove De Moivre's formula.

15. Find the limit of the sequence (z_n) where $z_n = \frac{2+in}{1+3n}$.

16. Find $\lim\limits_{z \to 2i} = z^2 - 2z + 1$.

17. Using C R equations, show that $f(z) = x^2 + y + i(y^2 - x)$ is not analytic at any point.

18. If $f(z)$ is an analytic function and $f'(z) = 0$ everywhere in the domain then prove that $f(z)$ is a constant in the domain.

19. Prove that $\sin z = 0$ if and only if $z = k\pi$.

20. Find dw/dz if $w = \exp[\sin 2z]$.

21. Prove the identity $\cosh^2 z - \sinh^2 z = 1$.

22. Find all values of $(-2)^i$.

23. Find the principal value of $(i)^{2i}$.

24. Show that $\int_0^\pi e^{it} dt = 2i$.

25. Find an upper bound for $\left| \int_C \frac{e^z}{z^2+1} dz \right|$ Where C is the circle $|z| = 2$.

26. Evaluate $\int_C \frac{1}{z} dz$ where C is the half circle in the counter clockwise direction joining $-i$ and i.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions.

27. Describe the set of points z satisfying $|z+2| = |z-1|$.

28. Write the quotient $\dfrac{1+i}{\sqrt{3}-i}$ in polar form.

29. Find all cube roots of $\sqrt{2} + i\sqrt{2}$.

30. If $\lim_{z \to z_0} f(z) = A$, $\lim_{z \to z_0} g(z) = B$, then show that $\lim_{x \to z_0} f(z)g(z) = AB$.

31. Express $f(z) = \dfrac{x-1-iy}{(x-1)^2 + y^2}$ in terms of z and $\bar{z}$.

32. Check the differentiability of $f(z) = \bar{z}$.

33. Prove that e^z is entire and find its derivative.

34. If $f(z) = u(x,y) + i\,v(x,y)$ is analytic in a domain D then prove that $u(x,y)$ and $v(x,y)$ are harmonic in D.

35. Find poles and their multiplicities for $f(z) = \dfrac{(3z+3i)(z^2-4)}{(z-2)(z^2+1)^2}$

36. Find $\lim_{z \to 0} \dfrac{\sin z}{z}$.

37. Show that $\sin^{-1}(z)$ is a multiple valued function and $\sin^{-1}(z) = -i\log\left[iz + \left(1-z^2\right)^{1/2}\right]$.

38. State deformation invariance theorem. Using this prove Cauchy's integral theorem.

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions.

39. (a) Compute $(1+i)^{24}$

(b) Find $\int_0^{2\pi} \cos^4 \theta \, d\theta$.

(c) Compute the derivative of $\left(\frac{z^2-1}{z^2+1}\right)^{100}$.

40. State and prove the necessary condition for a function to be differentiable at a point z_0.

41. (a) Construct an analytic function whose real part is $u(x,y) = x^3 - 3xy^2 + y$.

(b) Verify that $u(x,y) = e^x \sin y$ is harmonic in its region of definition.

42. (a) If $z \neq 0$, define $\log z$. Find $\log(1+i)$, $\log(-i)$.

(b) Determine the domain of analyticity for $f(z) = \log(3z - i)$. Compute $f'(z)$.

43. (a) Evaluate $\int_{Cr} (z - z_0)^n dz$ where Cr is the circle $|z - z_0| = r$.

(b) Let f be a continuous function in the domain D. If f has an antiderivative in D, prove that every loop integral of f in D vanishes.

44. (a) Evaluate $\oint_{|z|=2} \frac{e^z}{z^2 - 9} dz$.

(b) Determine the possible values of $\int_\Gamma \frac{1}{z-a} dz$ where Γ is any positively oriented simple closed contour not passing through a.

(2 × 15 = 30 Marks)